

# Coterminal & Reference Angles

Standard: NGSE9-12.FE.2

Explain how unit circle in coordinate plane enables the extension of trig functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise

EQ:

How are co-terminal and reference angles identified & sketched?

## I: Coterminal angles

- Angles are coterminal if they have the same initial side and same terminal side.

Positive

add  $360^\circ$  or  $2\pi$

Negative

subtract  $360^\circ$  or  $2\pi$

Example #1 - find positive & negative coterminal angle

$225^\circ$

pos.:  $225^\circ + 360^\circ = 585^\circ$

positive

neg.:  $225^\circ - 360^\circ = -135^\circ$

negative

Degree

Example #2:  $-285^\circ$

positive:  $-285 + 360^\circ = 75^\circ$

negative:  $-285^\circ - 360^\circ = -645^\circ$

Radian

Example #3: find coterminals for  $\frac{17\pi}{9}$

pos.:  $\frac{17\pi}{9} + \frac{2\pi}{1} \frac{(9)}{(9)} = \frac{17\pi}{9} + \frac{18\pi}{9} = \frac{35\pi}{9}$

neg:  $\frac{17\pi}{9} - \frac{2\pi}{1} \frac{(9)}{(9)} = \frac{17\pi}{9} - \frac{18\pi}{9} = -\frac{\pi}{9}$

Example #4: find coterminal angles  $-570^\circ$

positive:  $-570^\circ + 360^\circ = -210^\circ$  ~~still~~ *negative*

$-210^\circ + 360^\circ = 150^\circ$  ✓

negative:  $-570 - 360 = -930^\circ$

Are these angles coterminal?

$\frac{5\pi}{3}, -\frac{11\pi}{3}$

pos.  $\rightarrow$  neg

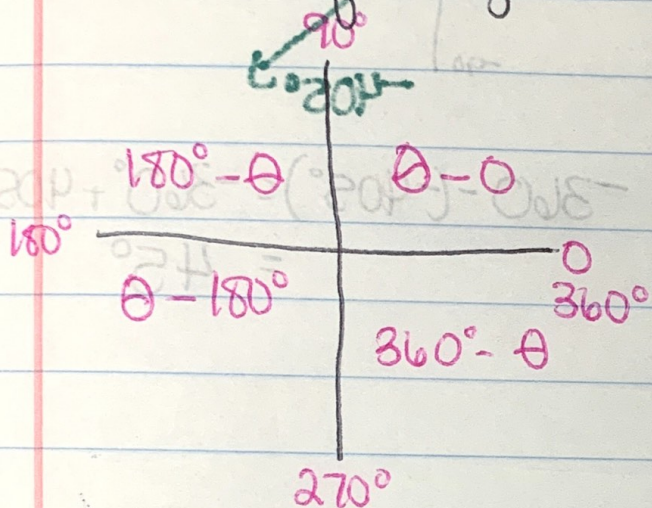
neg:  $\frac{5\pi}{3} - \frac{2\pi}{1} \frac{(3)}{(3)} = \frac{5\pi}{3} - \frac{6\pi}{3} = -\frac{\pi}{3}$  *too low*

$-\frac{\pi}{3} - \frac{2\pi}{1} \frac{(3)}{(3)} = -\frac{\pi}{3} - \frac{6\pi}{3} = -\frac{7\pi}{3}$  *too low again*

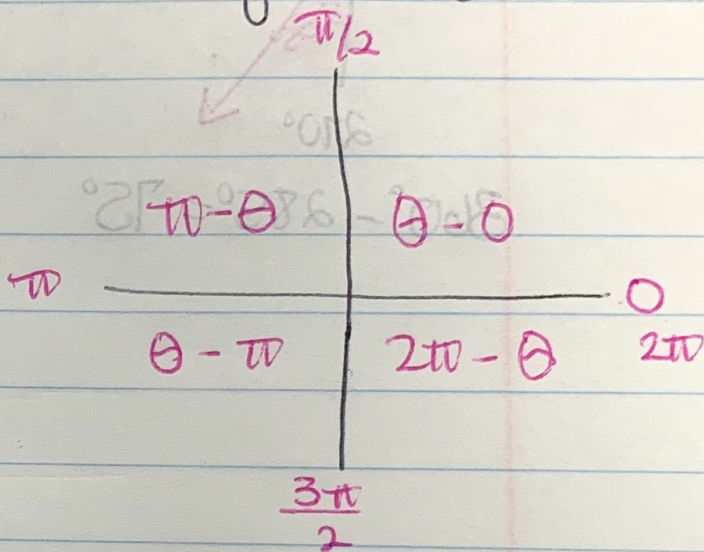
$$-\frac{7\pi}{3} - \frac{2\pi}{1} (3) = -\frac{7\pi}{3} - \frac{6\pi}{3} = -\frac{13\pi}{3} \rightarrow \text{no! Too high}$$

## II - Reference Angles

Rule for Degrees



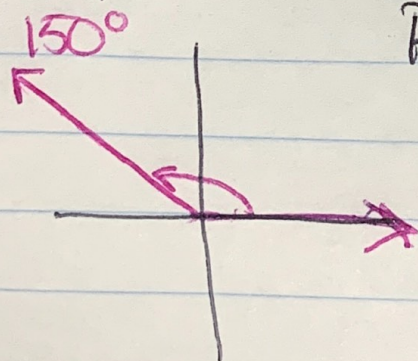
Rule for Radians



\*\* Quadrant I : IV flip when the angle is negative.

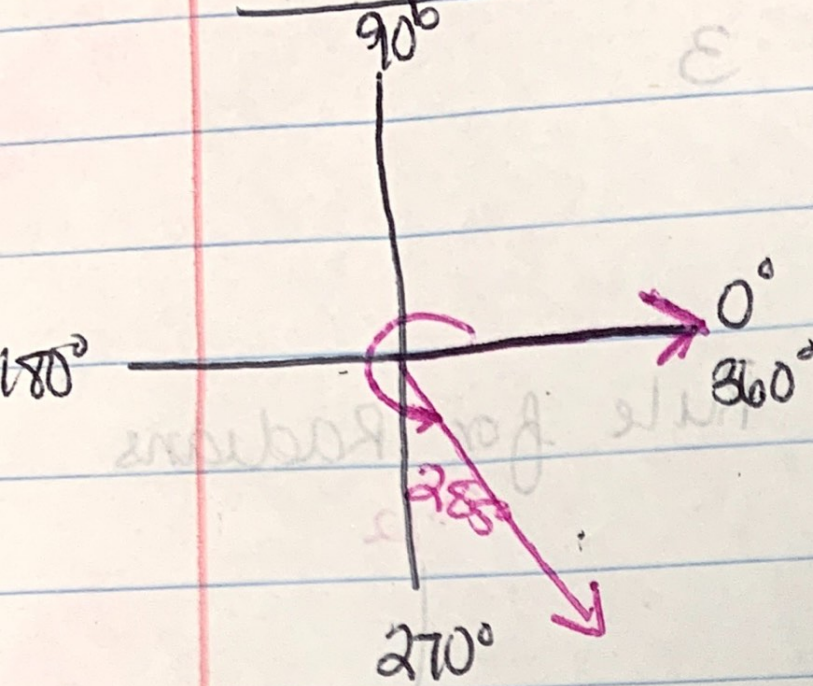
\*\*\* you're looking at distance between terminal side of angle : closest X-axis

Example 1 :



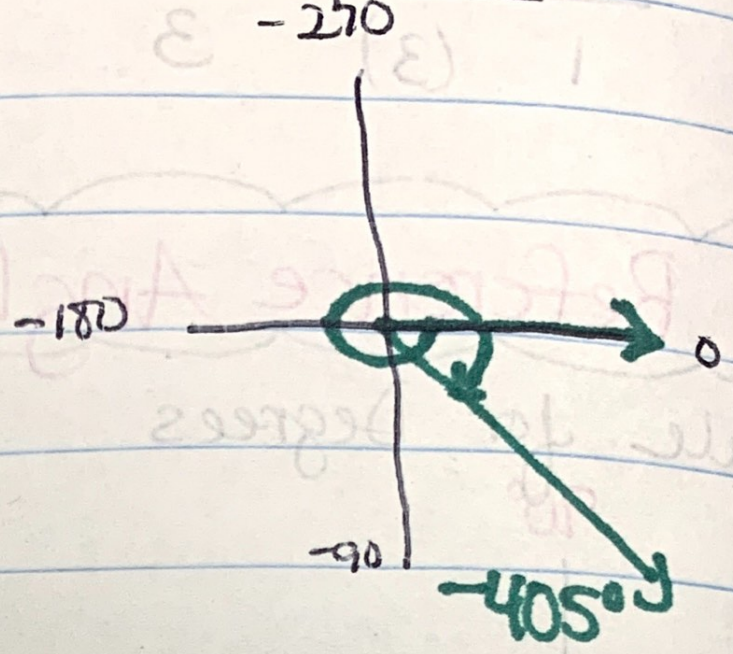
Reference angle:  
 $180^\circ - 150^\circ = 30^\circ$

Example 2:



$$360^\circ - 285^\circ = 75^\circ$$

Example #3



$$\begin{aligned} -360 - (-405^\circ) &= -360^\circ + 405^\circ \\ &= 45^\circ \end{aligned}$$