

Standard: MGSE9-12.G.SRT.11

Understand and apply Law of Sines and Cosines to find unknown measurements right and non-right triangles.

Topic: The Ambiguous Case

EQ: What is the ambiguous case and how is it applied to finding missing sides and angles?

Ambiguous Case - Law of Sines

Acute Angles

$$h = b \sin A$$

If _____ **Possible Triangles**

$$a < h \quad \text{none}$$

$$a = h \quad \text{one}$$

$$a > b \quad \text{one}$$

$$h < a < b \quad \text{two}$$

Obtuse Angles

If _____ **Possible Triangles**

$$a \leq b \quad \text{none}$$

$$a \geq b \quad \text{one}$$

What type of triangle is it? Possible Triangles?

$$A = 58^\circ, a = 4.5, \text{ and } b = 12.8$$

$$h = 12.8 (\sin 58^\circ) \\ = 10.85$$

- A. acute, none**
- B. acute, one**
- C. obtuse, none**
- D. obtuse, one**

What type of triangle is it? Possible Triangles?

$$A = 94^\circ, a = 14.6, \text{ and } b = 14.6$$

- A. acute, one**
- B. acute, none**
- C. obtuse, none**
- D. obtuse, one**

What type of triangle is it? Possible Triangles?

$$A = 58^\circ, a = 11.4, \text{ and } b = 12.8$$

- A. acute, one**
- B. acute, two**
- C. acute, none**
- D. obtuse, one**

$$h = 10.85$$
$$h < a < b$$
$$10.85 < 11.4 < 12.8$$

What type of triangle is it? Possible Triangles?

$$A = 110^\circ, a = 125, \text{ and } b = 100$$

- A. acute, one**
- B. obtuse, one**
- C. acute, none**
- D. obtuse, none**

Let's solve...why no solution?

Example 1: $A = 58^\circ, a = 4.5, \text{ and } b = 12.8$

$$h = 10.85$$

Let's solve...2 solutions (2 possible triangles)

1. Case #1 (use proportion and inverse for angle)

- find missing angle
- find missing sides

2. Case #2 -

- find missing angle
- find missing sides

Let's solve...How many possible triangles?

$$A = 58^\circ, a = 11.4, \text{ and } b = 12.8$$

1st \triangle

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 58^\circ}{11.4} = \frac{\sin B}{12.8}$$

$$\sin B = 12.8 \left(\frac{\sin 58^\circ}{11.4} \right)$$

$$\sin B = .95219$$

$$\sin^{-1}(.95219) = \underline{72.21^\circ} \leftarrow B$$

$$C = 180^\circ - (72.21 + 58^\circ)$$

$$= \underline{49.79^\circ}$$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{11.4}{\sin 58^\circ} = \frac{c}{\sin 49.79^\circ}$$

$$c = 10.266 \text{ ft}$$

2nd \triangle

$$180 - 72.21 = 107.79^\circ = B$$

$$180 - (58^\circ + 107.79) = 14.21^\circ = C$$

$$C \rightarrow \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c = 3.299$$

$$\text{or } 3.30$$

$$\text{or } 3.3$$

$$\text{or } 3.29$$

Your turn...find missing sides and angles - in notebook

$$A = 58^\circ, a = 4.5, \text{ and } b = 5$$

