Rational Root Test

rational roots must be in form : $\pm \left(\frac{\text{factors of } p}{\text{factors of } q}\right)$

Ex: $f(x) = 2x^3 - 3x^2 - 11x + 6$

(q = leading coefficient, p = constant)

(list factors of 6) 6: 1, 2, 3, 6

(" " of 2) 2: 1 2

possible rational roots: $\pm\left\{\frac{1}{1}, \frac{1}{2}, \frac{2}{1}, \frac{2}{2}, \frac{3}{1}, \frac{3}{2}, \frac{6}{1}, \frac{6}{2}\right\}$ (or rep. fract)

$= \pm\left\{1, \frac{1}{2}, 2, \cancel{2}, 3, \frac{3}{2}, 6, \cancel{3}\right\}$ ← Don't write twice

to find roots: plug into function & set 0

$f(1) = 2(1)^3 - 3(1)^2 - 11(1) + 6$

$= 2 - 3 - 11 + 6 = -12 + 6 = 6$ NO

$f(3) = 2(3)^3 - 3(3)^2 - 11(3) + 6$

$= 54 - 27 - 33 + 6 < 0$ yes!

$f(2) = 2(2)^3 - 3(2)^2 - 11(2) + 6$

$= 16 - 12 - 22 + 6 = -12$ NO

$f(-2) = 2(-2)^3 - 3(-2)^2 - 11(-2) + 6$

$= -16 - 12 + 22 + 6 = 0$ yes

$$\begin{array}{r|rrrr} 3 & 2 & -3 & -11 & 6 \\ & \downarrow & 6 & 9 & -6 \\ \hline -2 & 2 & 3 & -2 & 0 \\ & \downarrow & -4 & 2 & \\ \hline & 2 & -1 & 0 & \end{array} \quad (x-3) \quad (x-3)(x+2)$$

$$= (2x-1)(x-3)(x+2)$$

Ex. $g(x) = 3x^3 - 4x^2 - 17x + 6$

q $\qquad$ p

6: 1, 2, 3, 6

3: 1, 3

$$\frac{p}{q} = \pm \left\{ \frac{1}{1}, \frac{1}{3}, \frac{2}{1}, \frac{2}{3}, \frac{3}{1}, \frac{3}{3}, \frac{6}{1}, \frac{6}{3} \right\}$$

$$= \pm \left\{ 1, \frac{1}{3}, 2, \frac{2}{3}, 3, 6 \right\}$$

$f(1) = 3(1)^3 - 4(1)^2 - 17(1) + 6$

$= 3 - 4 - 17 + 6 = -18 + 6 = -12$ NO

$f(2) = 3(2)^3 - 4(2)^2 - 17(2) + 6$

$= 24 - 16 - 34 + 6 = -20$ NO

$f(3) = 3(3)^3 - 4(3)^2 - 17(3) + 6 = 0$ ✓

$f(6) = 3(6)^3 - 4(6)^2 - 17(6) + 6 =$ NO

$f(1/3) = 3(1/3)^3 - 4(1/3)^2 - 17(1/3) + 6 = 0$ ✓

$$\begin{array}{r|rrrr} \therefore 3 & 3 & -4 & -17 & 6 \\ & \downarrow & 9 & 15 & -6 \\ \hline & 3 & 5 & -2 & 0 \end{array} \quad (x-3)$$

$\frac{1}{3}$ | 3 5 -2
↓ 1 2
3 6 0 $(x-\frac{1}{3})$

$(3x+6)$

$= (x-3)(3x+6)(x-\frac{1}{3})$

If we have a rational # as a zero it must be in the list.